

To Advanced Placement Calculus Student,

I congratulate you on accepting the challenge of taking the Advanced Placement Calculus course at Oakbrook. I have prepared this packet to give you some information about the course and help you get ready for Calculus. The Advanced Placement Calculus course offers secondary school students the opportunity to pursue and receive credit for college-level course work at the secondary level. The curriculum that I will follow this year is the one that is recommended by The College Board. Their curriculum is based on the premise that college level material can be taught successfully to able and well-prepared secondary school students. We will cover all of the topics in the Calculus AB curriculum which corresponds to at least as much material as a standard first semester course of college Calculus. One of the focal points of the course is the preparation for the 3 ¼ hour Advanced Placement Calculus AB Examination given in May 2016. This is a national exam that you will be expected to take as one of the requirements of this course. There is a registration fee required of about \$95.00 that will be your responsibility. This fee will be collected in May.

The AP Exam is graded on a scale of one to five and the results are used by many colleges and universities for placement purposes. It is possible for you to gain college credit and/or advanced placement as a result of your grade on this test. The Advanced Placement Exam requires the use of a graphing calculator. Students have found it helpful to own a TI -89, but you can get by using the TI-83/84. AP Calculus is a rigorous and demanding course. You are among the best math students in the country. You should be proud of that fact and be willing to develop your talents to their fullest. My goal is to help you accomplish that development, but I can only be a guide. It is up to you to put forth the kind of consistent effort necessary to achieve your potential.

I have also included a homework assignment that is due on the first day of school. Have a wonderful and relaxing summer. I am looking forward to delving into the study and exploration of this branch of mathematics. Calculus truly is a fascinating course, and together we will appreciate its complexities, eloquence, and problem solving capabilities.

Sincerely, Mrs. Womick

If you purchased a TI-89 calculator, you can refer to the tutorial at <http://www.prenhall.com/divisions/esm/app/graphing/ti89/>.

Name _____

READ THE FOLLOWING DIRECTIONS CAREFULLY!

1. Before answering any questions read through the given notes and examples for each topic.
2. This packet is to be handed in on the first day of school **Tuesday, August 18th.**
3. **All work must be shown** in the packet OR on separate paper attached to the packet
4. **Student Choice:** You may pick 13 out of 16 of the sections to complete but you are responsible for knowing the contents of the entire packet.
5. Completion of this packet is worth a *quiz* grade.
6. All work must be organized and neatly written.

Formula Sheet

Reciprocal Identities:

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

Quotient Identities:

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities:

$$\sin^2 x + \cos^2 x = 1 \quad \tan^2 x + 1 = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

Double Angle Identities:

$$\begin{aligned} \sin 2x &= 2 \sin x \cos x & \cos 2x &= \cos^2 x - \sin^2 x \\ \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x} & &= 1 - 2 \sin^2 x \\ & & &= 2 \cos^2 x - 1 \end{aligned}$$

Logarithms:

$y = \log_a x$ is equivalent to $x = a^y$

Product property:

$$\log_b mn = \log_b m + \log_b n$$

Quotient property:

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

Power property:

$$\log_b m^p = p \log_b m$$

Property of equality:

then $m = n$

If $\log_b m = \log_b n$,

Change of base formula:

$$\log_a n = \frac{\log_b n}{\log_b a}$$

Fractional exponent:

$$\sqrt[b]{x^c} = x^{\frac{c}{b}}$$

Negative Exponents:

$$x^{-n} = 1/x^n$$

The Zero Exponent: $x^0 = 1$, for x not equal to 0.

Multiplying Powers

Multiplying Two Powers of the Same Base:

$$(x^a)(x^b) = x^{(a+b)}$$

Multiplying Powers of Different Bases:

$$(xy)^a = (x^a)(y^a)$$

Dividing Powers

Dividing Two Powers of the Same Base:

$$(x^a)/(x^b) = x^{(a-b)}$$

Dividing Powers of Different Bases:

$$(x/y)^a = (x^a)/(y^a)$$

Slope-intercept form: $y = mx + b$

Point-slope form: $y = m(x - x_1) + y_1$

Standard form: $Ax + By + C = 0$

Name: _____

Show all work – no credit will be awarded for answers missing appropriate work.
No calculators!

Section I: Algebra Review

1. Solve $xy + y + 1 = y$ for y .

2. Solve $\ln y = kt$ for y .

3. Solve $\ln(y-1) - \ln 2 = x + \ln x$ for y

4. Factor: $x^2(x-1) - 4(x-1)$

Simplify each expression.

5. $\frac{(x^2)^3 x}{x^7}$

6. $\sqrt{x} \cdot \sqrt[3]{x} \cdot x^{\frac{1}{6}}$

7. $\frac{5(x+h)^2 - 5x^2}{h}$

8. $\frac{\frac{1}{x} + \frac{4}{x^2}}{3 - \frac{1}{x}}$

Simplify, using factoring of binomial expressions. Leave answers in factored form.

Example:

$$\begin{aligned} \frac{(x+1)^3(4x-9) - (16x+9)(x+1)^2}{(x-6)(x+1)} &= \frac{(x+1)^2[(x+1)(4x-9) - (16x+9)]}{(x-6)(x+1)} \\ &= \frac{(x+1)^2(4x^2 - 5x - 9 - 16x - 9)}{(x-6)(x+1)} \\ &= \frac{(x+1)^2(4x^2 - 21x - 18)}{(x-6)(x+1)} \\ &= \frac{(x+1)^2(4x+3)(x-6)}{(x-6)(x+1)} \\ &= (x+1)(4x+3) \end{aligned}$$

9. $(x-1)^3(2x-3) - (2x+12)(x-1)^2$

10. $\frac{(x-1)^2(3x-1) - 2(x-1)}{(x-1)^4}$

Simplify by rationalizing the numerator.

Example:

$$\frac{\sqrt{x+4}-2}{x} = \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \frac{x+4-4}{x(\sqrt{x+4}+2)} = \frac{x}{x(\sqrt{x+4}+2)} = \frac{1}{\sqrt{x+4}+2}$$

11. $\frac{\sqrt{x+9}-3}{x}$

12. $\frac{\sqrt{x+h}-\sqrt{x}}{h}$

Solve each equation or inequality for x over the set of real numbers.

13. $2x^4 + 3x^3 - 2x^2 = 0$

14. $\frac{2x-7}{x+1} = \frac{2x}{x+4}$

15. $\sqrt{x^2 - 9} = x - 1$

16. $|2x - 3| = 14$

17. $x^2 - 2x - 8 < 0$ [Your answer should be interval(s)]

18. $\frac{3x+5}{(x-1)(x^4+7)} = 0$

Find the x and y intercepts for each.

19. $y = 2x - 5$

20. $y = x^2 + x - 2$

21. $y = x\sqrt{16 - x^2}$

22. $y^2 = x^3 - 4x$

FUNCTIONS

To evaluate a function for a given value, simply plug the value into the function for x .

Recall: $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read "f of g of x" Means to plug the inside function (in this case $g(x)$) in for x in the outside function (in this case, $f(x)$).

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find $f(g(x))$.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each.

1. $f(2) =$ _____ 2. $g(-3) =$ _____ 3. $f(t+1) =$ _____

4. $f[g(-2)] =$ _____ 5. $g[f(m+2)] =$ _____ 6. $[f(x)]^2 - 2g(x) =$ _____

Let $f(x) = \sin(2x)$ Find each exactly.

7. $f\left(\frac{\pi}{4}\right) =$ _____ 8. $f\left(\frac{2\pi}{3}\right) =$ _____

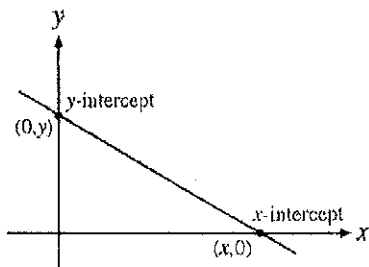
Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each.

9. $h[f(-2)] =$ _____ 10. $f[g(x-1)] =$ _____ 11. $g[h(x^3)] =$ _____

INTERCEPTS OF A GRAPH

To find the x-intercepts, let $y = 0$ in your equation and solve.

To find the y-intercepts, let $x = 0$ in your equation and solve.



Example: Given the function $y = x^2 - 2x - 3$, find all intercepts.

x-int. (Let $y = 0$)

$$0 = x^2 - 2x - 3$$

$$0 = (x - 3)(x + 1)$$

$$x = -1 \text{ or } x = 3$$

x-intercepts $(-1, 0)$ and $(3, 0)$

y-int. (Let $x = 0$)

$$y = 0^2 - 2(0) - 3$$

$$y = -3$$

y-intercept $(0, -3)$

Find the x and y intercepts for each.

12. $y = 2x - 5$

13. $y = x^2 + x - 2$

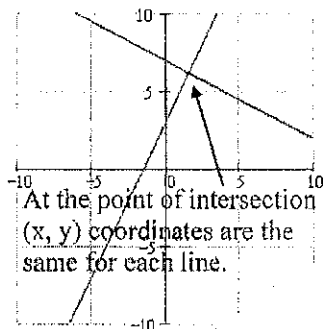
14. $y = x\sqrt{16 - x^2}$

15. $y^2 = x^3 - 4x$

POINTS OF INTERSECTION

Use substitution or elimination method to solve the system of equations.

Remember: You are finding a **POINT OF INTERSECTION** so your answer is an ordered pair.



CALCULATOR TIP

Remember you can use your calculator to verify your answers below. Graph the two lines then go to CALC (2nd Trace) and hit INTERSECT.

Example: Find all points of intersection of $x^2 - y = 3$
 $x - y = 1$

ELIMINATION METHOD

Subtract to eliminate y

$$x^2 - x = 2$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \text{ or } x = -1$$

Plug in $x = 2$ and $x = -1$ to find y

Points of Intersection: $(2, 1)$ and $(-1, -2)$

SUBSTITUTION METHOD

Solve one equation for one variable.

$$y = x^2 - 3$$

$$y = x - 1$$

Therefore by substitution $x^2 - 3 = x - 1$

$$x^2 - x - 2 = 0$$

From here it is the same as the other example

Find the point(s) of intersection of the graphs for the given equations.

16. $x + y = 8$
 $4x - y = 7$

17. $x^2 + y = 6$
 $x + y = 4$

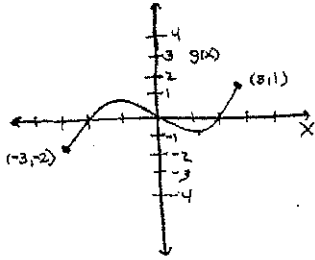
18. $x = 3 - y^2$
 $y = x - 1$

DOMAIN AND RANGE

Domain – All x values for which a function is defined (input values)

Range – Possible y or Output values

EXAMPLE 1



a) Find Domain & Range of $g(x)$.

The domain is the set of inputs for the function. Input values run along the horizontal axis. The furthest left input value associated with a pt. on the graph is -3 . The furthest right input values associated with a pt. on the graph is 3 . So Domain is $[-3, 3]$, that is all reals from -3 to 3 .

The range represents the set of output values for the function. Output values run along the vertical axis. The lowest output value of the function is -2 . The highest is 1 . So the range is $[-2, 1]$, all reals from -2 to 1 .

EXAMPLE 2

Find the domain and range of $f(x) = \sqrt{4-x^2}$
Write answers in interval notation.

DOMAIN

For $f(x)$ to be defined $4-x^2 \geq 0$.

This is true when $-2 \leq x \leq 2$

Domain: $[-2, 2]$

RANGE

The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0 .

Range: $[0, \infty)$

Find the domain and range of each function. Write your answer in INTERVAL notation.

19. $f(x) = x^2 - 5$

20. $f(x) = -\sqrt{x+3}$

21. $f(x) = 3 \sin x$

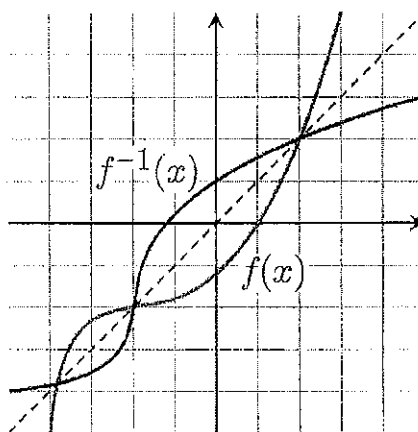
22. $f(x) = \frac{2}{x-1}$

INVERSES

To find the inverse of a function, simply switch the x and the y and solve for the new “ y ” value. Recall $f^{-1}(x)$ is defined as the inverse of $f(x)$

Example 1:

$f(x) = \sqrt[3]{x+1}$	Rewrite $f(x)$ as y
$y = \sqrt[3]{x+1}$	Switch x and y
$x = \sqrt[3]{y+1}$	Solve for your new y
$(x)^3 = (\sqrt[3]{y+1})^3$	Cube both sides
$x^3 = y+1$	Simplify
$y = x^3 - 1$	Solve for y
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation



Find the inverse for each function.

23. $f(x) = 2x + 1$

24. $f(x) = \frac{x^2}{3}$

25. $g(x) = \frac{5}{x-2}$

26. $y = \sqrt{4-x} + 1$

27. If the graph of $f(x)$ has the point $(2, 7)$ then what is one point that will be on the graph of $f^{-1}(x)$?

28. Explain how the graphs of $f(x)$ and $f^{-1}(x)$ compare.

EQUATION OF A LINE

Slope intercept form: $y = mx + b$

Vertical line: $x = c$ (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: $y = c$ (slope is 0)

* LEARN! We will use this formula frequently!

Example: Write a linear equation that has a slope of $\frac{1}{2}$ and passes through the point (2, -6)

Slope intercept form

$$y = \frac{1}{2}x + b$$

Plug in $\frac{1}{2}$ for m

$$-6 = \frac{1}{2}(2) + b$$

Plug in the given ordered

$$b = -7$$

Solve for b

$$y = \frac{1}{2}x - 7$$

Point-slope form

$$y + 6 = \frac{1}{2}(x - 2)$$

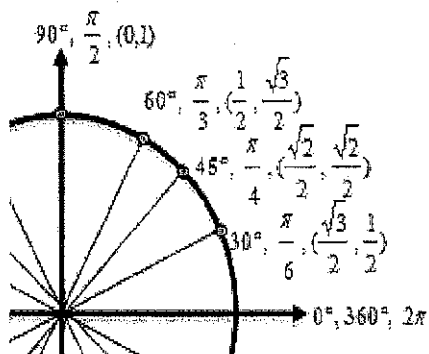
Plug in all variables

$$y = \frac{1}{2}x - 7$$

Solve for y

29. Determine the equation of a line passing through the point (5, -3) with an undefined slope.
30. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.
31. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of $\frac{2}{3}$.
32. Use point-slope form to find a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.
33. Use point-slope form to find a line perpendicular to $y = -2x + 9$ passing through the point (4, 7).
34. Find the equation of a line passing through the points (-3, 6) and (1, 2).
35. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3)

UNIT CIRCLE



You can determine the sine or the cosine of any standard angle on the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. Recall tangent is defined as \sin/\cos or the slope of the line.

Examples:

$$\sin \frac{\pi}{2} = 1 \quad \cos \frac{\pi}{2} = 0 \quad \tan \frac{\pi}{2} = \text{und}$$

***You must have these memorized OR know how to calculate their values without the use of a calculator.**

36. a.) $\sin \pi$ b.) $\cos \frac{3\pi}{2}$ c.) $\sin\left(-\frac{\pi}{2}\right)$ d.) $\sin\left(\frac{5\pi}{4}\right)$

e.) $\cos \frac{\pi}{4}$ f.) $\cos(-\pi)$ g.) $\cos \frac{\pi}{3}$ h.) $\sin \frac{5\pi}{6}$

i.) $\cos \frac{2\pi}{3}$ j.) $\tan \frac{\pi}{4}$ k.) $\tan \pi$ l.) $\tan \frac{\pi}{3}$

m.) $\cos \frac{4\pi}{3}$ n.) $\sin \frac{11\pi}{6}$ o.) $\tan \frac{7\pi}{4}$ p.) $\sin\left(-\frac{\pi}{6}\right)$

TRIGONOMETRIC EQUATIONS

Solve each of the equations for $0 \leq x < 2\pi$.

37. $\sin x = -\frac{1}{2}$

38. $2 \cos x = \sqrt{3}$

39. $4 \sin^2 x = 3$

**Recall $\sin^2 x = (\sin x)^2$

**Recall if $x^2 = 25$ then $x = \pm 5$

40. $2 \cos^2 x - 1 - \cos x = 0$ *Factor

TRANSFORMATION OF FUNCTIONS

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x - c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x + c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

41. Given $f(x) = x^2$ and $g(x) = (x-3)^2 + 1$. How does the graph of $g(x)$ differ from $f(x)$?

42. Write an equation for the function that has the shape of $f(x) = x^3$ but moved six units to the left and reflected over the x-axis.

43. If the ordered pair $(2, 4)$ is on the graph of $f(x)$, find one ordered pair that will be on the following functions:

a) $f(x) - 3$

b) $f(x - 3)$

c) $2f(x)$

d) $f(x - 2) + 1$

e) $-f(x)$

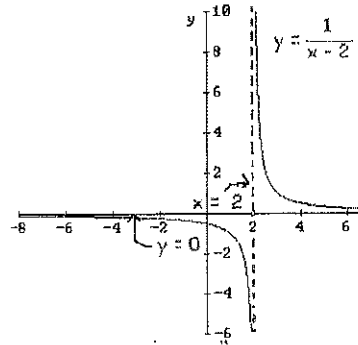
VERTICAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).

Write a vertical asymptotes as a line in the form $x =$

Example: Find the vertical asymptote of $y = \frac{1}{x-2}$

Since when $x = 2$ the function is in the form $1/0$ then the vertical line $x = 2$ is a vertical asymptote of the function.



44. $f(x) = \frac{1}{x^2}$

45. $f(x) = \frac{x^2}{x^2 - 4}$

46. $f(x) = \frac{2+x}{x^2(1-x)}$

47. $f(x) = \frac{4-x}{x^2 - 16}$

48. $f(x) = \frac{x-1}{x^2 + x - 2}$

49. $f(x) = \frac{5x+20}{x^2 - 16}$

HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Example: $y = \frac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Example: $y = \frac{2x^2 + x - 1}{3x^2 + 4}$ (As x becomes very large or very negative the value of this function will approach $2/3$). Thus there is a horizontal asymptote at $y = \frac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \frac{2x^2 + x - 1}{3x - 3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Determine all Horizontal Asymptotes.

50. $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

51. $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

52. $f(x) = \frac{4x^2}{3x^2 - 7}$

53. $f(x) = \frac{(2x-5)^2}{x^2 - x}$

54. $f(x) = \frac{-3x+1}{\sqrt{x^2+x}}$ * Remember $\sqrt{x^2} = \pm x$

This is very important in the use of limits.

EXPONENTIAL FUNCTIONS

Example: Solve for x

$$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$$

$$(2^2)^{x+1} = (2^{-1})^{3x-2} \quad \text{Get a common base}$$

$$2^{2x+2} = 2^{-3x+2} \quad \text{Simplify}$$

$$2x+2 = -3x+2 \quad \text{Set exponents equal}$$

$$x = 0 \quad \text{Solve for x}$$

Solve for x:

$$55. 3^{3x+5} = 9^{2x+1}$$

$$56. \left(\frac{1}{9}\right)^x = 27^{2x+4}$$

$$57. \left(\frac{1}{6}\right)^x = 216$$

LOGARITHMS

The statement $y = b^x$ can be written as $x = \log_b y$. They mean the same thing.

REMEMBER: A LOGARITHM IS AN EXPONENT

Recall $\ln x = \log_e x$

The value of e is 2.718281828... or $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$

Example: Evaluate the following logarithms

$$\log_2 8 = ?$$

In exponential form this is $2^? = 8$

Therefore $? = 3$

Thus $\log_2 8 = 3$

Evaluate the following logarithms

$$58. \log_7 7$$

$$59. \log_3 27$$

$$60. \log_2 \frac{1}{32}$$

$$61. \log_{25} 5$$

$$62. \log_9 1$$

$$63. \log_4 8$$

$$64. \ln \sqrt{e}$$

$$65. \ln \frac{1}{e}$$

PROPERTIES OF LOGARITHMS

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

$$\log_b x^y = y \log_b x$$

$$b^{\log_b x} = x$$

Examples:

Expand $\log_4 16x$

$$\log_4 16 + \log_4 x$$

$$2 + \log_4 x$$

Condense $\ln y - 2 \ln R$

$$\ln y - \ln R^2$$

$$\ln \frac{y}{R^2}$$

Expand $\log_2 7x^5$

$$\log_2 7 + \log_2 x^5$$

$$\log_2 7 + 5 \log_2 x$$

Use the properties of logarithms to evaluate the following

66. $\log_2 2^5$

67. $\ln e^3$

68. $\log_2 8^3$

69. $\log_3 \sqrt[5]{9}$

70. $2^{\log_2 10}$

71. $e^{\ln 8}$

72. $9 \ln e^2$

73. $\log_9 9^3$

74. $\log_{10} 25 + \log_{10} 4$

75. $\log_2 40 - \log_2 5$

76. $\log_2 (\sqrt{2})^5$

EVEN AND ODD FUNCTIONS

Recall:

Even functions are functions that are symmetric over the y-axis.

To determine algebraically we find out if $f(x) = f(-x)$

*(*Think about it what happens to the coordinate $(x, f(x))$ when reflected across the y-axis*)*

Odd functions are functions that are symmetric about the origin.

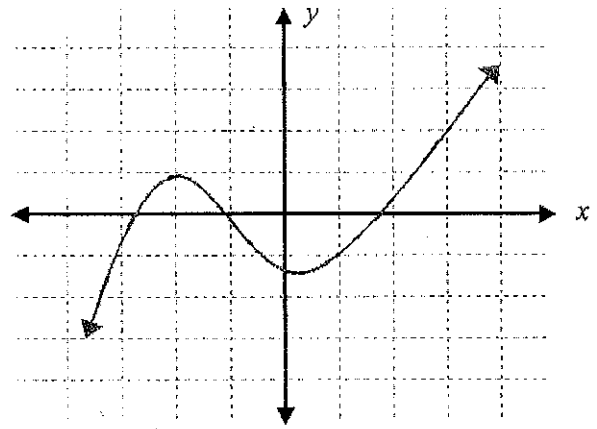
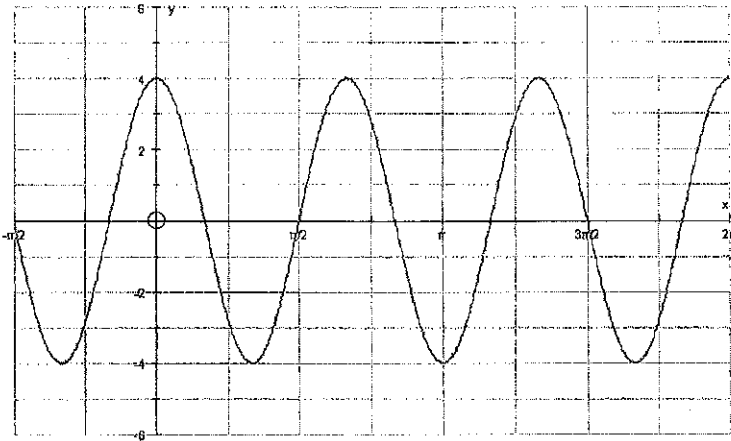
To determine algebraically we find out if $f(-x) = -f(x)$

*(*Think about it what happens to the coordinate $(x, f(x))$ when reflected over the origin*)*

State whether the following graphs are even, odd or neither, show ALL work.

77. _____

78. _____



79. _____

$$f(x) = 2x^4 - 5x^2$$

80. _____

$$g(x) = x^5 - 3x^3 + x$$

81. _____

$$h(x) = 2x^2 - 5x + 3$$

82. _____

$$j(x) = 2 \cos x$$

83. _____

$$k(x) = \sin x + 4$$

84. _____

$$l(x) = \cos x - 3$$

Fill in The Unit Circle

Positive:
Negative:

Positive:
Negative:

